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Alternative Policy Rankings in a Large, Open Economy with Sector-Specific, Minimum Wages*

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The analysis by international trade theorists of factor market imperfections, and alternative policy rankings in the presence thereof, has distinguished between two major, polar types: (i) a distortionary wage differential between two sectors, while the wage is perfectly flexible in each sector; and (ii) a sticky (or minimum) wage which is equal, however, between the two sectors.

The analysis of the former class of distortions was pioneered by Hagen [7] and has subsequently been extensively explored by Bhagwati and Ramaswami [3], Kemp and Negishi [9], and Bhagwati, Ramaswami and Srinivasan [2].

The analysis of the second class of distortions was pioneered by Gottfried Haberler [6] and has subsequently been extended fully for the traditional two-sector model of trade theory by Brecher [5].

The purpose of this paper is to analyze policy rankings in the presence of a yet different type of factor market imperfection, introduced in a pioneering paper by Harris and Todaro [8] which *combines* specificity of wages (in one sector) with a (*resulting*) wage differential between the two sectors in an ingenious manner. In earlier papers [3, 4], we have analyzed the Harris–Todaro model, for this range of issues, in the context of a closed economy or a "small," open economy with given terms of trade. In this paper, we analyze alternative policy rankings in the context

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of the fully general assumption of a "large" country, which has monopoly power in trade.

Section 1 outlines the model. Section 2 briefly outlines the principal results of our analysis. Section 3 analyzes the policy instrument defined by a wage subsidy in the sector with minimum wages. Section 4 discusses the policy instrument defined by a production tax-cum-subsidy. Section 5 analyzes the policy instrument defined by a consumption tax-cum-subsidy. Section 6 discusses a tariff policy. Finally, Section 7 derives the combination of policies yielding the first-best optimum.

1. The Model

The basic Harris–Todaro model consists of a set of relations which can be stated as follows.

There are two commodities (A and M), produced in quantitites X_A and X_M, using L_A and L_M units of labor, with strictly concave production functions. (Thus, implicitly, there is a second factor (K_A, K_M) which yields the diminishing returns to labor input.)

$$X_A \leqslant f_A(L_A), \tag{1}$$

$$X_M \leqslant f_M(L_M). \tag{2}$$

Next, with the fixed, overall labor supply assumed by choice of units to equal unity, we have

$$L_A + L_M \leqslant 1, \tag{3}$$

$$L_A, L_M \geqslant 0. \tag{4}$$

We now introduce foreign trade. Let E denote net exports of the agricultural good, exchanging for $g(E)$ of net imports of manufacturing. (Since we do not wish to prejudge the question as to which commodity will be imported, E is allowed to take on negative values as well, in which case $g(E)$ will also be negative. In such a case, agricultural goods will be imported and manufactured goods will be exported.) We further assume that $g(0) = 0, g' > 0, g'' < 0$. This implies that the marginal (g') and average (g/E) terms of trade decline as E increases and the marginal is less than the average. The domestic consumption of the two commodities will then be

$$C_A = X_A - E, \tag{5}$$

$$C_M = X_M + g(E). \tag{6}$$

It is well known that if we now add a standard utility function

$$U = U[C_A, C_M], \tag{7}$$

where U is concave with positive marginal utilities for finite $[C_A, C_M]$, neoclassical free trade equilibrium will be characterized by

$$U_1/U_2 = f_M'/f_A', \tag{8}$$

$$U_1/U_2 = g(E)/E, \tag{9}$$

together with (1)–(6) being satisfied (where U_1 and U_2 represent the partial derivatives of U with respect to C_A and C_M, respectively, and f_i' is the derivation of f_i with respect to its argument, $i = A, M$). (We rule out corner solutions by assuming $f_A'(0) = f_M'(0) = \infty$.)

Figure 1 shows the production possibility curve BJ and the foreign offer curve PDC superimposed on it at P a la Baldwin. At the production

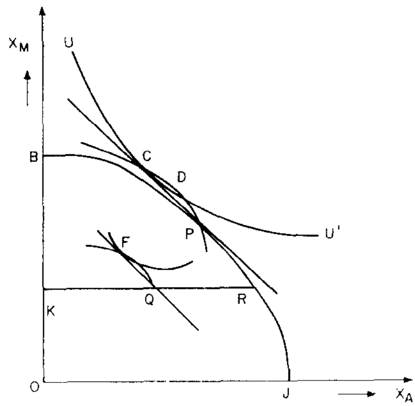

FIGURE 1

point P, the price ratio faced by producers (i.e., the negative of the slope of PC) is the same as the marginal rate of substitution in production as represented by the (negative of the) slope of the tangent to the production possibility curve at P. At the consumption point C, this price ratio equals the marginal rate of substitution in consumption as represented by the (negative of the) slope of the indifference curve at C. (The production and consumption points are so located that the price ratio equals the external terms of trade. The curve PDC corresponds to the graph of $g(E)$.)

The Harris–Todaro problem of sector-specific rigid wages and resulting unemployment can now be readily introduced. Let the free trade solution above (at P and C in Fig. 1) be $X_A^*, X_M^*, L_A^*, L_M^*\ (=1 - L_A^*)$, E^*, C_A^*, C_M^*. Assume now, however, that there is an exogenously specified, minimum wage constraint in manufacturing, such that

$$w \geqslant \bar{w}, \tag{10}$$

where w is the wage in manufacturing, in units of the manufacturing good (M). For a competitive economy, this implies that

$$f_M'(L_M) \geqslant \bar{w}. \tag{10'}$$

This constraint becomes binding, and P in Fig. 1 is inadmissible, when

$$f_M'(L_M^*) < \bar{w}.$$

The competitive economy, when characterized by this wage constraint, will then experience unemployment of labor. We then have two options to characterize the labor market equilibrium in this situation: *either* assume that the wage in agriculture (A) will be equalized with the wage in manufacturing (M) despite the unemployment; *or* that the wage in agriculture will be equalized with the *expected* wage in manufacturing, the expected and the actual wage in manufacturing being different as the former would be defined, as the latter weighted by the rate of employment, i.e.

$$\bar{w}L_M/(1 - L_A),$$

where $L_M < (1 - L_A)$ when there is unemployment.

The analysis of Harris–Todaro is based on the latter assumption, so that we can then write the equilibrium production conditions in competition and laissez-faire, as follows.

$$f_M' = \bar{w}, \tag{11}$$

$$(U_1/U_2)f_A' = \bar{w}L_M/(1 - L_A), \tag{12}$$

$$U_1/U_2 = g(E)/E. \tag{13}$$

We assume, in (12), that the production and consumption prices for the agricultural good are the same. (In writing Eq. (11), we assume that the producer and consumer prices of the manufacturing good are identical, with wage $\bar{w}$ paid in kind. Hence, the effect of a production subsidy to manufacturing is essentially not to affect any real decisions, as those made *via* equations (11) and (12), but merely to increase each commodity price in terms of the (arbitrary) unit of account. However, if we were to assume instead that the producer and consumer prices of the manufacturing good could be made to differ by policy, then the worker in manufacturing would earn the value of his marginal product at the producer price and then, *qua* consumer, must have enough income (in terms of the unit of account) to buy $\bar{w}$ units of the manufacturing good. In that case, a wage subsidy policy to manufacturing would be equivalent to a production subsidy policy to manufacturing, as is the case in the agricultural sector. Thus, note that, if we did shift to the latter, alternative assumption on wage payment in the manufacturing sector, then the analysis would not change but our policy equivalences would. In particular, the first best optimal policy mix would then include a uniform production subsidy to both sectors, and a production subsidy in manufacturing and a wage subsidy in agriculture.) Given $\bar{w}$, we can solve (11), (12) and (13) for L_M, L_A, and E after setting $X_A = f_A(L_A)$, $X_M = f_M(L_M)$, $C_A = f_A(L_A) - E$ and $C_M = f_M(L_M) + g(E)$. The equilibrium production point corresponding to this situation of nonintervention, with unemployment, will then lie, in Fig. 1, along RK (where X_M and hence, L_M are fixed at the value that makes $f_M' = \bar{w}$) at Q. (It is worth noting that the nonintervention equilibrium would lie along RK even if we assumed actual wages to be equalized between the two sectors.) The consumption point will be at F.

The policy question that emerges then, is: What alternative policies can be used in this model for intervention and what would be their impact on welfare and on unemployment?

2. The Basic Results

In this model, there are a number of policy options which can be explored; however, many can be shown to be equivalent to one another or to combinations of other policies.

Thus, we will discuss the following policies: (i) nonintervention or laissez-faire; (ii) wage subsidy in manufacturing (M); (iii) production subsidy to agriculture (A); and (iv) consumption subsidy to agriculture (A).

Note that, as a little reflection will show, the simple structure of the model implies that: (v) a wage subsidy in agriculture is equivalent to policy (iii); (vi) a uniform wage tax-cum-subsidy in all employment is a combination of policies (ii) and (iii); and (vii) a tariff policy is a combination of policies (iii) and (iv).

We will proceed to establish the following propositions.

THEOREM 1. *There exists a unique equilibrium corresponding to each wage subsidy s to manufacturing in an interval* $[0, \bar{s}]$. *At $\bar{s}$, full employment is reached.*

THEOREM 2. *A wage subsidy (in manufacturing) will exist which will improve welfare over laissez-faire.*

Thus, laissez-faire (i.e., wage subsidy = 0) can be necessarily improved upon by some wage subsidy. In fact, any positive subsidy in some interval with zero as its left end point will be welfare-improving.

THEOREM 3. *The full-employment wage subsidy $\bar{s}$ may not be the "second-best" wage subsidy and may be inferior even to laissez-faire.*

THEOREM 4. *There exists a unique production subsidy which will enable full employment to be reached and which is also the second-best production subsidy.*

THEOREM 5. *The second-best wage subsidy (to manufacturing) and production subsidy (to agriculture) cannot be ranked uniquely.*

THEOREM 6. *There exists a unique consumption subsidy which will enable full employment to be reached and which is also the second-best consumption subsidy.*

THEOREM 7. *The second-best wage subsidy (to manufacturing) and the second-best consumption subsidy (to agriculture) cannot be ranked uniquely.*

THEOREM 8. *The second-best production and consumption subsidies cannot be ranked uniquely.*

THEOREM 9. *A tariff (or trade subsidy) policy may not improve welfare but can improve employment.*

THEOREM 10. *The first-best optimum can be reached if, in addition to the monopoly-power-in-trade tariff, a combination of a production tax-cum-subsidy and wage subsidy (to manufacturing) or any equivalent thereof (including a uniform wage subsidy on employment of labor in both sectors), is provided.*

The combination of a suitable production subsidy to agriculture plus an appropriate wage subsidy in manufacturing, or its equivalents, such as a uniform wage subsidy in all employment, will yield the first-best optimum, when also combined with an appropriate tariff to exploit the postulated monopoly power in trade (as discussed in Section 7).

3. Wage Subsidy in Manufacturing

Let us now consider the wage subsidy as the policy intervention in this economy. Denoting by s the subsidy per unit of labor employed in manufacturing, we find that the equilibrium is now characterized by

$$f_M' = \bar{w} - s, \tag{14}$$

$$(U_1/U_2) f_A' = \bar{w} L_M/(1 - L_A), \tag{15}$$

$$(U_1/U_2) = g(E)/E. \tag{16}$$

Equation (14) assumes that each worker in manufacturing receives remuneration $\bar{w}$, of which only $(\bar{w} - s)$ is paid by the employer and s by the state out of some form of nondistortionary taxation. With the consumer and producer price of the agricultural good assumed to be identical, and equal to U_1/U_2, we then have the actual wage in agriculture being equated to the employment-rate-weighted (i.e., expected) wage in manufacturing in Eq. (15).

Existence of equilibrium is established once we show that values of L_A, L_M, and E exist that satisfy (14)–(16) and the conditions that (i) L_A and L_M are nonnegative and their sum does not exceed the available labor force, namely, unity; and (ii) the value of E is such that whichever commodity is exported, the volume of exports does not exceed production. We now proceed to show that, in fact, unique values of L_A, L_M and E exist that satisfy all the above conditions. In doing so, we shall use, in addition to the assumptions already made, the assumption that both goods are normal in consumption.

Denoting the average terms of trade $g(E)/E$ by $\phi(E)$, we see that our assumptions on g imply that $\phi > 0$, $\phi' < 0$, and $g' < \phi$ for all E. Substituting (16) into (15) we get

$$\phi(E) f_A' = \bar{w} L_M/(1 - L_A). \tag{15'}$$

Given $\bar{w}$, s, concavity of f_M, and the assumption that $f_M' \to \infty$ as $L_M \to 0$, Eq. (14) uniquely determines L_M as a function $L_M(s)$ of s for all s in $0 \leqslant s \leqslant \bar{w}$. Given the value $L_M(s)$ for L_M, the range of feasible values of L_A is $[0, 1 - L_M(s)]$. For any value of L_A in this range, the feasible values of E are confined to the interval $[g^{-1}\{-f_M(L_M(s))\}, f(L_A)]$, the reason being that if A is exported the volume of exports E cannot exceed the production $f(L_A)$ and if A is imported, then $-E$ is the *value* of exports of M. The physical volume of exports of M is $-g(E)$ and this cannot exceed the production $f_M(L_M(s))$. Thus $-g(E) \leqslant f_M(L_M(s))$ or

$$E \geqslant g^{-1}\{-f_M(L_M(s))\},$$

where g^{-1} is the inverse function of g. We first show that given any feasible L_A there exists a unique, feasible E that satisfies (16). We then substitute this value of E (denoting it $E(L_A, s)$ to indicate its dependence on the specified values of L_A and s) into the left-hand side of (15′) and show that a unique feasible value of L_A satisfies (15′).

Now consider (16). The right-hand side is a decreasing function of E while the left-hand side is an increasing function of E for given values of L_A and $L_M(s)$, since

$$\frac{\partial}{\partial E}\left(\frac{U_1}{U_2}\right) = \frac{(-U_{11} + U_{12}g')\,U_2 - (-U_{21} + U_{22}g')\,U_1}{U_2^2} > 0$$

by virtue of the facts that $g' > 0$ and the normality assumptions ensure that $U_{11}U_2 - U_{21}U_1 > 0$ and $U_{22}U_1 - U_{12}U_2 < 0$. Further, as E approaches its lowest feasible value, the volume of exports of the manufactured good approaches its production; the result is that its domestic consumption C_M approaches zero. Similarly, as E approaches its highest feasible value, the domestic consumption C_A of the agricultural good approaches zero. Now, if we assume that the marginal utility $U_1(U_2)$ of agricultural good (manufactured good) tends to ∞ as its consumption $C_A(C_M)$ tends to zero, the left-hand side of (16) increases from zero to $+\infty$ as E increases from its lower to upper limiting value and hence, given s, for any feasible L_A there exists a unique E denoted by $E(L_A, s)$ which satisfies (16).

It is easily seen that $\partial E(L_A, s)/\partial L_A > 0$. For, given s (and hence, X_M) and a feasible E (and hence, C_M), C_A increases as L_A increases resulting in a decrease in U_1/U_2 (given our assumption of normality for both goods). Thus, as L_A increases, the graph of the left-hand side of (16) shifts to the right while the graph of the right-hand side stays put, resulting in a larger value for the E at which the two graphs intersect. The reader can readily verify, using a similar argument, that $\partial E/\partial s < 0$.

Let us now substitute the function $E(L_A, s)$ for E in (15′). Then, for any given s, both sides of (15′) are functions of L_A only. The left-hand side of (15′) is then a decreasing function of L_A since

$$(\partial/\partial L_A)\{\phi f_A'\} = \phi'(\partial E/\partial L_A) + \phi f_A'' < 0$$

because $\phi > 0$, $\phi' < 0$, $\partial E/\partial L_A > 0$, and $f_A'' < 0$. The right-hand side is an increasing function of L_A. Further, as $L_A \to 0$, the left-hand side (i.e., $\phi f_A'$) also $\to \infty$, and hence, exceeds the right-hand side which takes the value $\bar{w}L_M(s)$. Hence, if we show that as $L_A \to$ its maximum feasible value $1 - L_M(s)$, the left-hand side is less than the right-hand side, we would have shown the existence of a unique feasible L_A satisfying (15′).

Consider $s = 0$. Then $L_M(0)$ satisfies $f_M' = \bar{w}$. By assumption, $L_M{}^*$ (the laissez-faire value of L_M without the minimum wage constraint) results in $f_M' < \bar{w}$ and hence, $L_M^* > L_M(0)$. This means that $L_A^* = 1 - L_M^* < 1 - L_M(0)$. Thus if we set $L_A = 1 - L_M(0)$, its maximum feasible value given $s = 0$, the following hold true:

(i) $f_A'(1 - L_M(0)) < f_A'(L_A^*)$ (concavity of f_A) ;
(ii) $E\{1 - L_M(0), 0\} > E\{(1 - L_M^*), 0\}$ (since $\partial E/\partial L_A > 0$) ;
(iii) $\phi[E\{1 - L_M(0), 0\}] < \phi[E\{1 - L_M^*, 0\}]$ (since $\phi' < 0$).

Thus, $\phi f_A'$ (left-hand side of (15′)) evaluated at the largest feasible value of L_A (given $s = 0$), i.e., at $1 - L_M(0)$, is less than its value evaluated at $L_A = 1 - L_M^*$. But at $L_A = 1 - L_M^*$, $\phi f_A' = (U_1/U_2) f_A' = f_M' < \bar{w}$. Hence, a fortiori, the value of $\phi f_A'$ at $L_A = 1 - L_M(0)$ is less than $\bar{w}$. This in turn implies that, for $s = 0$, the graphs of the two sides of (15′) intersect at a unique L_A between zero and $1 - L_M(0)$, as shown in Fig. 2.

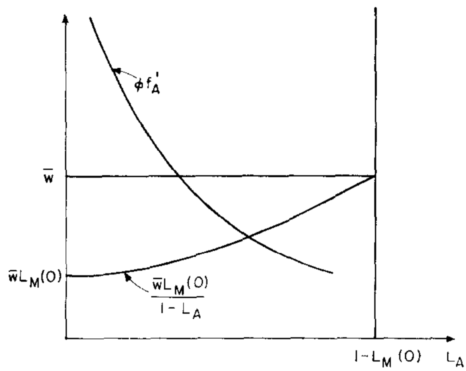

FIGURE 2

Thus we have established the existence of a unique laissez-faire equilibrium with unemployment under the minimum wage constraint.

Existence of Unique Equilibrium for Each Value of s in $[0, \bar{s})$

Now, as s is increased, for any given L_A the left-hand side of (15′) increases, since $(\partial/\partial s)(\phi f_A') = \phi' f_A'(\partial E/\partial s) > 0$, and hence, its graph shifts to the right. The right-hand side also increases since $L_M(s)$ increases with s. Thus, its graph shifts to the left, with its value at $L_A = 1 - L_M(s)$ always equal to $\bar{w}$. Hence, the two graphs continue to intersect at a unique L_A in the interval $\{0, 1 - L_M(s)\}$ as s increases up to a maximum value $\bar{s}$, when this value of L_A equals its upper bound $1 - L_M(\bar{s})$, and full employment is reached. This is shown in Fig. 3. For values of $s > \bar{s}$, no equilibrium exists. Thus, we have shown the existence of a unique equilibrium for each value of s in $[0, \bar{s}]$.

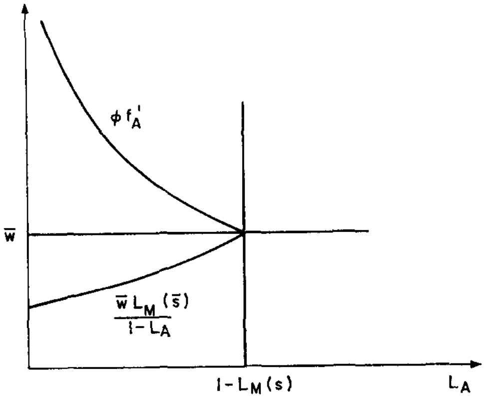

FIGURE 3

Impact on Welfare of Change in s

Let us now evaluate the change in welfare, i.e., dU/ds, as s increases. After some manipulation, the following can be derived,

$$\frac{dE}{ds} = -\left[\left\{\phi f_A'' - \frac{\bar{w} L_M}{(1 - L_A)^2}\right\} \frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) + \frac{\bar{w}}{1 - L_A} \frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right)\right] \Big/ D f_M'', \tag{17}$$

$$\frac{dL_A}{ds} = \left[\phi' f_A' \frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) + \left\{\frac{\partial}{\partial E}\left(\frac{U_1}{U_2}\right) - \phi'\right\} \frac{\bar{w}}{(1 - L_A)}\right] \Big/ D f_M'', \tag{18}$$

$$\frac{dU}{ds} = -U_2 \frac{N}{D f_M''}, \tag{19}$$

where

$$D = \phi' f_A' \frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) + \left\{\phi f_A'' - \frac{\bar{w} L_M}{(1 - L_A)^2}\right\}\left\{\phi' - \frac{\partial}{\partial E}\left(\frac{U_1}{U_2}\right)\right\}, \tag{20}$$

$$\begin{aligned} N = {} & \left[-\phi \left\{\phi f_A'' - \frac{\bar{w} L_M}{(1 - L_A)^2}\right\} - \phi\phi'(f_A')^2 - \frac{\bar{w}\phi f_A' g'}{(1 - L_A) f_M'}\right] \\ & \times \frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) \\ & + \left[\frac{g'\bar{w}}{(1 - L_A)} + \frac{f_M'}{f_A'}\left\{\phi f_A'' - \frac{\bar{w} L_M}{(1 - L_A)^2}\right\} + f_M' f_A' \phi'\right] \frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) \\ & + \phi' f_M' \phi f_A'' + \frac{s\bar{w} L_M \phi'}{(1 - L_A)^2}. \end{aligned} \tag{21}$$

Now, if we assume normality of both goods in consumption, then

$$\frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) > 0, \qquad \frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) < 0,$$

$$\frac{\partial}{\partial E}\left(\frac{U_1}{U_2}\right) = -\frac{1}{f_A'}\frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) + \frac{g'}{f_M'}\frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) > 0.$$

Further, $\phi > 0$, $\phi' < 0, f_A' > 0, f_A'' < 0, f_M' > 0$ and $U_2 > 0$. Hence, $D > 0$. It is seen that $dE/ds < 0$, i.e., the net export of the agricultural commodity decreases as the wage subsidy to manufacturing increases. However, the signs of dL_A/ds and dU/ds are in general indeterminate. But, using the fact that the marginal terms of trade g' is by assumption less than the average terms of trade ϕ, we can show that

$$N > \left[-\phi^2 f_A'' - \phi\phi'(f_A')^2 - \frac{s\phi\bar{w}L_M}{(1-L_A)^2 f_M'}\right]\frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right)$$

$$+ \left[f_M' f_A' \phi' + \frac{\phi f_M' f_A''}{f_A'} + \frac{s\bar{w}L_M}{(1-L_A)^2 f_A'}\right]\frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right)$$

$$+ \phi' f_M' \phi f_A'' + \frac{s\bar{w}L_M\phi'}{(1-L_A)^2}.$$

In the above inequality, all terms involving s explicitly are negative and the rest are positive. When $s = 0$, the terms involving s drop out making N and hence, $dU/ds > 0$ at $s = 0$. By continuity this means that welfare can be increased over its laissez-faire level by giving any positive wage subsidy in an interval. It is also clear that the full-employment wage subsidy need not be the second-best optimum subsidy.

4. Production Subsidy

We now consider the policy of subsidizing production in agriculture. To do this, we rewrite the critical equilibrium conditions as follows:

$$f_M' = \bar{w}, \tag{22}$$

$$\pi_p f_A' = \bar{w}L_M/(1 - L_A), \tag{23}$$

$$U_1/U_2 = \phi(E), \tag{24}$$

where π_p is the producer's price of the agricultural good, the production subsidy being $(\pi_p - \phi)/\phi$ per unit.

Now (22) determines L_M uniquely as $L_M(0)$ (its laissez-faire value). The feasible values of L_A then lie in the interval $[0, 1 - L_M(0)]$. Equation (24) is the same as (16) when $s = 0$ and hence, for any feasible L_A, there exists a unique $E(L_A, 0)$ which satisfies (24) and clearly $\partial E/\partial L_A > 0$. Now the left-hand side of (23) is a decreasing function of L_A (for any given π_p) and the right-hand side is an increasing function of L_A. We have already seen that when π_p is at its laissez-faire value, the graphs of the two sides intersect at a unique $L_A(0)$ such that $0 < L_A(0) < 1 - L_M(0)$. Now, as we increase π_p continuously above its laissez-faire value, thus increasing the rate of production subsidy, the graph of the left-hand side of (23) shifts to the right and continues to intersect the right-hand side (which does not shift) at a feasible value of L_A until π_p reaches a value $\bar{\pi}_p$ at which the intersection occurs at $L_A = 1 - L_M(0)$. At this point, full employment is attained; and for values of $\pi_p > \bar{\pi}_p$, no equilibrium exists.

It is also clear that as π_p increases, L_A increases and hence, X_A increases, i.e., $dL_A/d\pi_p > 0$ and $dX_A/d\pi_p = f_A'(dL_A/d\pi_p) > 0$. It can thus be shown that

$$\frac{dU}{d\pi_p} = U_2 \left[\frac{g' \left\{ -\frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) + \frac{\phi f_A'}{f_M'} \frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) \right\} - \phi\phi' f_A'}{-\frac{1}{f_A'}\frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) + \frac{g'}{f_M'}\frac{\partial}{\partial L_M}\left(\frac{U_1}{U_2}\right) - \phi'} \right] \frac{dL_A}{d\pi_p}$$
$$> 0. \tag{25}$$

Hence, clearly the second-best optimum production subsidy is the full-employment subsidy (which is the maximum, feasible subsidy).

5. Consumption Subsidy

We now consider the policy of subsidizing the consumption of agricultural goods. To do this, we must rewrite the equilibrium conditions as follows,

$$f_M' = \bar{w}, \tag{26}$$

$$\phi(E) f_A' = \bar{w}\, L_M/(1 - L_A), \tag{27}$$

$$\pi_c = U_1/U_2, \tag{28}$$

where π_c is the consumer's price of agricultural good, the consumption subsidy being $(\phi(E) - \pi_c)/\pi_c$ per unit.

Now, consider (28). For any given L_A, the right-hand side is an increasing function of E. Further, as E tends to its lower limiting value of

$g^{-1}\{-f_M(L_M(0))\}$, U_1/U_2 tends to zero; and as E tends to its upper limiting value of $f_A(L_A)$, $U_1/U_2 \to \infty$. Hence, for any positive π_c, there exists a unique E denoted by $E(L_A, \pi_c)$ that satisfies (28). It is also clear that $\partial E(L_A, \pi_c)/\partial L_A > 0$ and $\partial E(L_A, \pi_c)/\partial \pi_c > 0$.

Substituting $E(L_A, \pi_c)$ for E in (27), we find that, for a given π_c, the left-hand side of (27) is a decreasing function of L_A while the right-hand side is an increasing function.

We have already seen (in Section 3) that when π_c equals its laissez-faire value, the graph of the two sides of (27) will intersect at a unique $L_A(0)$, satisfying $0 < L_A(0) < 1 - L_M(0)$. Furthermore, as we decrease π_c, thus increasing the rate of consumption subsidy, the graph of the left-hand side will shift to the right, while the graph of the right-hand side stays put. Hence, until π_c reaches a value $\bar{\pi}_c$, the two graphs will intersect at a feasible value of L_A; and at $\bar{\pi}_c$, they will intersect at $L_A = 1 - L_M(0)$. For any lower value of π_c, there is no equilibrium.

It is also obvious that the equilibrium value of L_A (and hence, X_A) increases as π_c decreases, i.e., $dL_A/d\pi_c < 0$ and $dX_A/d\pi_c < 0$. It can thus be shown that

$$\frac{dE}{d\pi_c} = \left[\frac{\bar{w}L_M}{(1 - L_A)^2} - \phi f_A''\right] \frac{dL_A}{d\pi_c} \Big/ f_A'\phi' > 0, \tag{29}$$

$$\frac{dU}{d\pi_c} = U_2 \left[\phi f_A' \frac{dL_A}{d\pi_c} + (g' - \phi) \frac{dE}{d\pi_c}\right] < 0 \qquad (\text{since } g' < \phi). \tag{30}$$

This means that, as π_c decreases from its laissez-faire value to its full-employment value $\bar{\pi}_c$, welfare increases. Thus, the full employment subsidy is also the second-best consumption subsidy.

6. Trade Tariff (Subsidy)

Let us now consider a tariff policy. The equilibrium will now be characterized by

$$f_M' = \bar{w}, \tag{31}$$

$$\phi(E)(1 + t) f_A' = \bar{w}L_M/(1 - L_A), \tag{32}$$

$$U_1/U_2 = \phi(E)(1 + t), \tag{33}$$

where t is the ad valorem tariff rate. If the agricultural commodity is exported (imported), i.e., E is positive (negative), then t represents an export subsidy (import duty).

As earlier, L_M is uniquely determined at $L_M(0)$ by (31). From the argument of Section 3, it follows that for any given t and L_A in the feasible range $\{0, 1 - L_M(0)\}$, there exists a unique feasible $E(L_A, t)$ that satisfies (33). It is also clear that

$$\frac{\partial E}{\partial L_A} = \frac{-(\partial/\partial L_A)(U_1/U_2)}{(\partial/\partial E)(U_1/U_2) - \phi'(1+t)} > 0,$$

$$\frac{\partial E}{\partial t} = \frac{\phi}{(\partial/\partial E)(U_1/U_2) - \phi'(1+t)} > 0.$$

Substituting $E(L_A, t)$ for E in (32), we then see that the left-hand side is a decreasing function of L_A while the right-hand side is an increasing function of L_A. We know that, when $t = 0$, the graphs of the two sides intersect at a unique $L_A(0)$ in $\{0, 1 - L_M(0)\}$. As we increase t above zero, the graph of the left-hand side shifts to the right while that of the right-hand side stays put, so that the two graphs continue to intersect at an L_A in the feasible range *until* t reaches a value $\bar{t}$ when the intersection occurs at $L_A = 1 - L_M(0)$, thereby attaining full employment. For $t > \bar{t}$, there is no equilibrium.

Furthermore, as t increases, equilibrium L_A increases. It can then be shown that

$$\frac{dL_A}{dt} = \frac{\phi f_A'(\partial/\partial E)(U_1/U_2)}{\left[\begin{array}{r}\left\{\phi'(1+t) - \dfrac{\partial}{\partial E}\left(\dfrac{U_1}{U_2}\right)\right\}\left\{\phi(1+t)f_A'' - \dfrac{\bar{w}L_M(0)}{(1-L_A)^2}\right\} \\ + f_A'\phi'(1+t)\dfrac{\partial}{\partial L_A}\left(\dfrac{U_1}{U_2}\right)\end{array}\right]} > 0, \tag{34}$$

$$\frac{dE}{dt} = \phi\left[-f_A'\frac{\partial}{\partial L_A}\left(\frac{U_1}{U_2}\right) + \frac{\bar{w}L_M(0)}{(1-L_A)^2} - \phi(1+t)f_A''\right] \times \frac{dL_A}{dt}\Big/ f_A'\frac{\partial}{\partial E}\left(\frac{U_1}{U_2}\right) > 0, \tag{35}$$

$$\begin{aligned}\frac{dU}{dt} &= U_1\left\{f_A'\frac{dL_A}{dt} - \frac{dE}{dt}\right\} + U_2\left\{f_M'\frac{dL_M}{dt} + g'\frac{dE}{dt}\right\} \\ &= U_2\left[f_A'\phi(1+t)\frac{dL_A}{dt} + \{g' - \phi(1+t)\}\frac{dE}{dt}\right].\end{aligned} \tag{36}$$

Now, (36) shows clearly that the change in welfare dU/dt is the sum of two terms consisting of a production effect $U_2 f_A'\phi(1+t)(dL_A/dt)$ and a consumption and trade effect $U_2\{g' - \phi(1+t)\}(dE/dt)$. The production effect is unambiguously positive. Since $U_2 > 0$ and $dE/dt > 0$, the sign of the trade effect depends on that of $g' - \phi(1+t)$. By assumption, $g' < \phi$

and hence, $g' - \phi(1 + t) < -t\phi$. For nonnegative values of t the consumption effect is therefore negative while for negative values of t it depends on whether g' exceeds or falls short of $\phi\,(1 + t)$. Thus we cannot assert anything *in general* about the welfare effect of a tariff. However, as we said earlier, L_A and hence, total employment $L_A + L_M(0)$ increases monotonically as the tariff is increased and full employment is reached at $\bar{t}$.

7. Optimal Policy Intervention

We may now briefly state the combination of policies which would yield the first-best optimum in this model.

Thus, let t^* be the optimal tariff and s^* the optimal wage subsidy in all employment, which would obtain at the optimal equilibrium. We would then be meeting the constraints of the model as follows.

$$f_M' = \bar{w} - s^*, \tag{37}$$

$$\phi(E)(1 + t^*)f_A' = \bar{w} - s^*, \tag{38}$$

$$\phi(E)(1 + t^*) = U_1/U_2\,, \tag{39}$$

and

$$g'(E) = U_1/U_2 = f_M'/f_A'. \tag{40}$$

The diagrammatic counterpart of this optimal equilibrium is shown in Fig. 4, where the optimal wage subsidy is supposed, along with the optimal

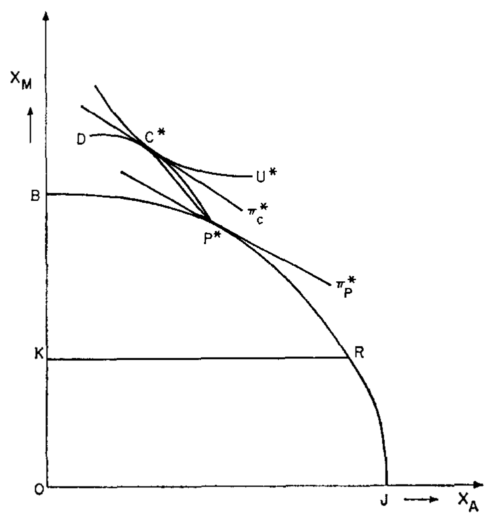

Figure 4

tariff, to lead to production at P^* (tangent to production price-ratio $\pi_p{}^*$), consumption at C^* (tangent to identical consumption price-ratio $\pi_c{}^* = \pi_p{}^* = \phi(E)(1 + t^*)$) and international terms of trade $\phi(E)$ equal to P^*C^*. The utility function is then maximized at value U^*.

It is readily seen, of course, that the uniform wage subsidy s^* could be given equivalently as wage subsidy to manufacturing alone, as rate s^*, *plus* a suitable production subsidy to agriculture, and so on. To derive other equivalences, the reader can refer to our earlier discussion of this subject in Section 2.

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